



Sensor Network Design and Upgrade for Plant Parameter Estimation

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Abstract

This work concentrates on the comparison of two approaches for the design of sensor networks with parameter estimation purposes. After extending a Maximum Precision Model recently published to multiple parameter estimation and binary variables, its equivalence with the Minimum Cost Model is presented. Industrial heat exchanger units are used to illustrate the results.

Keywords: Sensor network design, Parameter estimation, Data reconciliation, Observability

Introduction

The availability of reliable process knowledge is essential to parameter estimation. This information is obtained through monitoring and data reconciliation only when an adequate set of instruments has been located at the right places. The measurement arrangement should guarantee the observability and precision of the variables involved in the estimation scheme. Furthermore, the assessment of cost optimal measurement structures for performance estimation is a challenging issue for complex plants.

Several authors have addressed the problem of selecting measurement structures to determine accurate parameter values. Furthermore, several works have appeared in the literature to design sensor networks for steady state process. The designs satisfied different purposes, such as observability, precision, cost, reliability and robustness. A survey of the state of the art can be found in Bagajewicz (1997). Among these strategies, we are concerned with Maximum Precision Models and Minimum Cost Models for parameter estimation.

The incorporation of measurements to observable systems in order to simultaneously minimize additional cost and error estimates has been addressed by Kretsovalis and Mah (1987). Madron and Veverka (1992) obtained a set of sensors that minimizes a measure of all the error estimates after reconciliation, without considering sensor costs.

Later on, Alheritiere et al. (1997) presented the optimization of the existing resources allocated to the sensors in an industrial site, for improving the accuracy of one parameter. In this formulation, neither binary variables nor multiple parameter estimation are included in the analysis.

In Bagajewicz (1997) a MINLP problem is proposed to obtain Minimum Cost sensor structures for linear systems subject to constraints on precision and robustness. Robustness measures are Gross error detectability, Precision availability and Resilience.

The analysis of the existing NLP Maximum Precision Model reveals some limitations in their application, which motivates the development of a more comprehensive model for this approach. This work proposes a MINLP problem that overcomes the detected limitations by using binary variables and inequality cost constraints. Both the design and upgrade models are proposed. Furthermore, the mathematical connection between Minimum Cost Models with precision constraints (Bagajewicz, 1997) and the new Maximum Precision Model will be shown. In particular, it will be shown when these two models provide the same solution. The comparison between both approaches will be done using industrial heat exchanger units.

Parameter sensitivities have a fundamental role in the resolution of both types of problems, thus, the importance of redundancy in the accuracy of estimation procedures for the design stage will also be shown.

Generalized Maximum Precision Model

Maximum Precision Models include all those models developed for sensor network design that contain a measure of the estimation quality of parameters or state variables in the objective function. In this area, Alheritiere et al. (1997) proposed a NLP problem to obtain the optimal redistribution of fixed resources to the different sensors of an existing plant, in order to maximize the precision of a parameter estimate. Data reconciliation is not applied in their procedure.

This type of NLP is not appropriate from the point of view of sensor network design and upgrade because a) the continuous representation of variables leads to non-discrete values for the number of sensors b) different sets of measurements can lead to the estimation of the same parameter. The selected procedure chooses one a priori c) If a set of measurements leading to the estimation of the parameter is redundant, a smaller variance for each variable can be obtained using data reconciliation. Therefore fewer measurements can accomplish the same parameter variance.

In this section a generalized maximum precision model for design and upgrade of sensor networks is presented, that avoids previous drawbacks. A design model considers the minimization of a weighted sum of the standard deviation (σ_j^2) of the parameters after data reconciliation, constrained by a bound on the capital expenditure (c_T)

$$\left. \begin{array}{l} \text{Min} \quad \sum_{j \in M_p} a_j \sigma_j^2(q) \\ \text{s.t.} \quad \sum_{i \in M_l} c_i q_i \leq c_T \\ q_i = 0,1 \quad \forall i \in M_l \end{array} \right\} \quad (1)$$

where M_p is the set of parameters that is desired to estimate, M_l is the set of allowed locations of measurements and q_i is a binary variable. Hardware redundancy and the availability of different types of instruments may be incorporated to the previous formulation. The upgrading of a sensor network by the addition of instrumentation can be accomplished using the model represented by:

$$\left. \begin{array}{l} \text{Min} \quad \sum_{j \in M_p} a_j \sigma_j^2(q^N, q^E) \\ \text{s.t.} \quad \sum_{i \in M_n} \sum_{k=1}^{K_n} c_{ik} q_{ik}^N \leq c_T \\ \sum_{k=1}^{K_n} q_{ik}^N + \sum_{k=1}^{K_e} q_{ik}^E \leq N_i^* \quad N_i^* \geq 0 \\ q_{ik}^N \in [0,1,2,\dots] \quad \forall i,k \in Mn \\ q_{ik}^E \in [1,2,\dots] \quad \forall i,k \in Me \end{array} \right\} \quad (2)$$

where M_n = set of variables that may be measured, q_{ik}^N = number of sensors of type k added for the measurement of variable i , K_n = number of different types of sensors available to measure i , M_e = set of variables already measured, q_{ik}^E = number of sensors of type k already installed to measure variable i , c_{ik} = cost of a new sensor to measure variable i of type k , K_e = number of different types of existing sensors to measure i , N_i^* = maximum number of sensor that may be used to measure a variable i

For a selected set of instruments represented by the vector q , the standard deviation $\sigma_j(q)$ may be estimated by the following three-step procedure: a) Model Linearization b) Observability Analysis c) Calculation of parameter estimates standard deviations.

The generalized models have many advantages over the NLP formulation: a) they can provide a design or upgrade for multiple parameter estimation b) Since binary variables are used, more realistic results in accordance with the discrete nature of sensors can be obtained, as opposed to non-discrete number of sensors given by NLP formulations c) they take into account redundancy and all possible forms of obtaining the parameters, not just one.

Minimum Cost Model

Given a set of parameters that is desired to estimate M_p , the selection of instruments such that the cost is minimized and accuracy constraints on parameters are satisfied, involves the solution of the following optimization problem

$$\left. \begin{array}{l} \text{Min} \quad \sum_{i \in M_l} c_i q_i \\ \text{s.t.} \quad \sigma_j(q) \leq \sigma_j^* \quad \forall j \in M_p \\ q_i = 0,1 \quad \forall i \in M_l \end{array} \right\} \quad (3)$$

For simplicity, in the formulation of the objective function it is assumed that there is only one potential measuring device with associated cost c_i for each variable and hardware redundancy is not considered. This formulation is only a special case of a more general problem, which involves the accurate and robust estimation of a set of variables M , containing parameters and state variables (Bagajewicz, 1997). If hardware redundancy and different types of instruments are considered, the minimum cost model for instrumentation upgrading is represented by equation (4). Variable nomenclature has been previously defined.

$$\left. \begin{array}{l}
 \text{Min} \sum_{i \in M_n} \sum_{k=1}^{K_n} c_{ik} q_{ik}^N \\
 \text{s.t.} \\
 \sigma_j(q) \leq \sigma_j^* \quad \forall j \in M_p \\
 \sum_{k=1}^{K_n} q_{ik}^N + \sum_{k=1}^{K_e} q_{ik}^E \leq N_i^* \quad N_i^* \geq 0 \\
 q_{ik}^N \in [0,1,2] \quad \forall i,k \in M_n \\
 q_{ik}^E \in [1,2] \quad \forall i,k \in M_e
 \end{array} \right\} \quad (4)$$

$$\left\{ \begin{array}{l}
 c_T = \alpha \leq \text{Min } f(q) = \text{Min} \sum_{i \in M_1} c_i q_i \\
 \text{s.t.} \\
 g(q) = - \sum_{j \in M_p} a_j \sigma_j^2 \geq - \sum_{j \in M_p} a_j (\sigma_j^*)^2 = \beta \\
 \sigma_j(q) \leq \sigma_j^* \quad \forall j \in M_p \\
 q_i = 0,1 \quad \forall i \in M_1
 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l}
 \beta = - \sum_{j \in M_p} a_j (\sigma_j^*)^2 \geq \text{Max } g(q) = \text{Min} \sum_{j \in M_p} a_j \sigma_j^2 \\
 \text{s.t.} \\
 f(q) = \sum_{i \in M_1} c_i q_i \leq c_T = \alpha \\
 \sigma_j(q) \leq \sigma_j^* \quad \forall j \in M_p \\
 q_i = 0,1 \quad \forall i \in M_1
 \end{array} \right\} \quad (5)$$

Example

To illustrate the duality, let us consider the following example. The heat transfer coefficients for a set of three heat exchangers, where crude is heated up using hot gas-oil coming from a column, are estimated in terms of temperature and flow rate measurements (see Figure 1, Table 4).

Flowmeters of 3% precision are installed on streams 1, 5 and 7. All streams have a thermocouple, which standard deviation is 2°F, except 1, where hardware redundancy of two is available. The standard deviations of heat transfer coefficients calculated using the installed set of instruments are [8.09 4.76 8.59] after data reconciliation.

In order to enhance parameter precision's, new instruments should be added. In this example, hardware redundancy is considered. New flowmeters of precision=3% and cost=\$2250 are available. Two types of temperature sensors of 2°F and 0.2°F of standard deviation may be selected for streams 1,4 and 9, costs are \$500 and \$1500 respectively. Only instruments of 2°F of precision are allowed for all the other streams.

There are restrictions for instrument location. The maximum number of allowed instruments for measuring the flowrate and temperature of streams are given by vectors $N_r^* = [1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1]$ and $N_T^* = [2 \ 1 \ 1 \ 2 \ 1 \ 1 \ 1 \ 0 \ 2]$, where a zero indicates no instruments can be installed to measure the variable. In this example, weights in the objective function are selected equal to one.

Tables 1, 2 and 3 present results for both types of models when data reconciliation is applied to estimate the parameters. When there are two possible

Duality of Sensor Network Models

There is a mathematical connection between the Maximum Precision Model, given by (1) and the Minimum Cost Model given by (3). If both models are modified by adding trivial constraints as in (5), it may be shown that the Minimum Cost Model is the dual of the Maximum Precision Model in the Tuy sense. This duality was established in general by Tuy (1987). Thus, relationship (5) is applied in our case:

instruments to measure a variable, the type of instrument is indicated between parenthesis in the optimal solution set.

As it is shown in these tables, the optimal cost of instrumentation increases with the precision of the parameters' estimates and system redundancy. For some cases, no set of available instruments can fulfill precision requirements.

Furthermore, some examples show the duality between the Generalized Maximum Precision Model and the Minimum Cost Model. These are Cases 1 and 2, Cases 3 and 5 from Tables 1 and Tables 2 and 3 respectively.

Conclusions

Various models for the design and upgrade of instrumentation for parameter estimation have been discussed. Maximum Precision models and Minimum Cost models have been presented and the mathematical relationship between them has been established. The drawbacks of existing NLP models were avoided using these MINLP models.

References

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Table 1: Results for the Minimum Cost Model

Case #	$\sigma_{U_1}^*$	$\sigma_{U_2}^*$	$\sigma_{U_3}^*$	σ_{U_1}	σ_{U_2}	σ_{U_3}	C_{\min}	Optimal Set(s)
1	3.5	2.5	3.5	2.6636	1.6873	2.3797	500	T ₄ (1)
				2.6633	1.6416	2.6368		T ₉ (1)
2	2.5	1.5	2.5	-	-	-	-	-
3	2.6	1.5	2.2	2.5936	1.4903	2.0903	6500	F ₂ F ₃ T ₄ (2) T ₉ (1)
								F ₂ F ₄ T ₄ (2) T ₉ (1)
								F ₃ F ₄ T ₄ (2) T ₉ (1)
4	3.0	1.5	2.5	2.5947	1.4911	2.2828	5500	F ₂ F ₃ T ₄ (1) T ₉ (1)
								F ₂ F ₄ T ₄ (1) T ₉ (1)
								F ₃ F ₄ T ₄ (1) T ₉ (1)

Table 2: Results for the Maximum Precision Model

Case #	$\sigma_{U_1}^*$	$\sigma_{U_2}^*$	$\sigma_{U_3}^*$	c_T	σ_{U_1}	σ_{U_2}	σ_{U_3}
1	3.5	2.5	3.5	10000	2.5784	1.4474	2.0673
2	3.5	2.5	3.5	500	2.6636	1.6873	2.3797
3	2.5	1.5	2.5	20000	-	-	-
4	2.6	1.5	2.2	20000	2.5668	1.4296	2.0505
5	2.6	1.5	2.2	6500	2.5936	1.4903	2.0903

Table 3: Results for the Maximum Precision Model (Continued)

Case #	c	$\sum_i \sigma_{\eta_i}^2$	Optimal Set(s)
1	9750	13.017	F ₂ F ₃ F ₄ T ₄ (2) T ₉ (2)
2	500	15.605	T ₄ (1)
3	-	-	-
4	12000	12.837	F ₂ F ₃ F ₄ F ₉ T ₄ (2) T ₉ (2)
5	6500	13.317	F ₃ T ₄ (2) T ₉ (1) F ₂ F ₄ T ₄ (2) T ₉ (1) F ₃ F ₄ T ₄ (2) T ₉ (1)

Table 4: Data for the Heat Exchanger Network

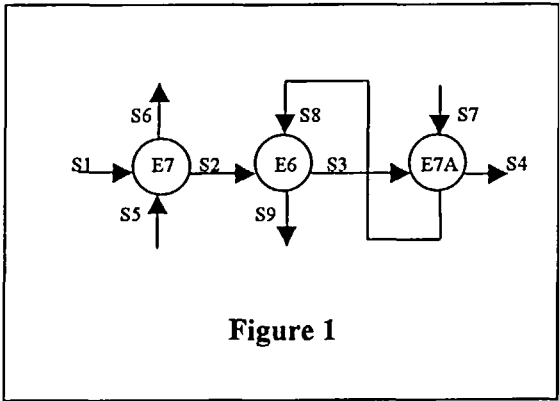


Figure 1

Stream	Flowrate (Mlb)	Temperature (F)
S1	224.677	542.854
S2	224.677	516.295
S3	224.677	448.279
S4	224.677	402.219
S5	217.019	307.637
S6	217.019	339.806
S7	398.008	191.182
S8	398.008	221.636
S9	398.008	266.876

Area E7=500 ft² Area E6=1100 ft²
Area E7A=700 ft²